

Semi-dileptonic decays of the light vector mesons in Light Front Quark Model

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Abstract

We study the transition form factors of the light vector to pseudoscalar mesons as functions of the momentum transfer q^2 within the light-front quark model. With these form factors, we calculate the decay branching ratios of all possible modes for $V \rightarrow P\ell^+\ell^-$ ($V = \omega$ and ϕ , $P = \pi^0$, η and η' and $\ell = e$ and μ). We find that our numerical results fit with the data, such as those of $\omega \rightarrow \pi^0\ell^+\ell^-$ and $\phi \rightarrow \pi^0e^+e^-$ by NA60 and $\phi \rightarrow \eta e^+e^-$ by SND. We also predict that the branching ratios of $\phi \rightarrow \pi^0\mu^+\mu^-$, $\omega \rightarrow \eta e^+e^-$, $\omega \rightarrow \eta\mu^+\mu^-$, $\phi \rightarrow \eta\mu^+\mu^-$ and $\phi \rightarrow \eta'e^+e^-$ to be around 3.48×10^{-6} , 3.22×10^{-6} , 1.81×10^{-9} , 6.86×10^{-6} 2.97×10^{-7} , respectively.

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I. INTRODUCTION

The investigations of the transition processes between light vectors ($V = \omega$ and ϕ) and pseudoscalars ($P = \pi^0$, η and η') mesons are helpful to infer the internal physics properties of the mesons, in particular, the non-perturbative QCD effects. These processes can be described by transition form factors, denoted as $f_{V \rightarrow P}$, from the parametrizations of the hadronic matrix elements. Some non-perturbative QCD approaches are available to evaluate these elements, such as the lattice QCD, QCD sum rule and vector mesons dominant (VMD). Phenomenologically, the relativistic light front quark model (LFQM) also provides a convenient method to study these form factors.

Motivated by the recent accurate data on $\omega \rightarrow \pi^0 \mu^+ \mu^-$ by the NA60 collaboration [1], we would like to study the transition form factors of $f_{V \rightarrow P}$ within the framework of the LFQM. In particular, we concentrate on the decay processes of $V \rightarrow P \ell^+ \ell^-$ with $V = (\omega, \phi)$, $P = (\pi^0, \eta, \eta')$ and $\ell = (e, \mu)$. In the LFQM, one can calculate the form factors in the frame where the momentum transfer is purely longitudinal, *i.e.*, $p_\perp = 0$ and $p^2 = p^+ p^-$, which covers the whole allowed kinematic region of $0 \leq p^2 \leq p_{\text{max}}^2$. We will use the phenomenological light front (LF) meson wave functions [2–4] to evaluate $f_{V \rightarrow P}$ in the LFQM. The LF wave functions can be constructed by the simplest structures of the meson constituents in terms of quark-antiquark ($Q\bar{Q}$) Fock states [4], which have been widely applied to study the form factors of the Dalitz decays [2, 4, 5].

The paper is organized as follows: In Sec. II, we present our formalism for the transition form factors of $V \rightarrow P$ within the LFQM and the decay branching ratios of $V \rightarrow P \ell^+ \ell^-$. In Sec. III, we perform our numerical calculations on these processes. We will also compare our results in the LFQM with the data as well as other theoretical predictions. We give our conclusions in Sec. IV.

II. FORMALISM

We start with the decay process

$$V(p_v) \rightarrow P(p_p) \gamma^*(q) \rightarrow P(p_p) \ell^+(p_1) \ell^-(p_2), \quad (1)$$

where V represents the vector meson of ω or ϕ , P stands for the pseudoscalar of π^0 or η or η' and $\ell = e$ or μ , while p_v , p_p , q , p_1 and p_2 are the corresponding momenta. The reason that

we only consider the process in Eq. (1) is that the dominant contributions to the dilepton channels of the vector mesons are through the exchanges of the virtual photon. The decay amplitude for the process in Eq. (1) is given by [6]:

$$A = ie^2 f_{V \rightarrow P}(q^2) \varepsilon_{\mu\nu\rho\sigma} \epsilon^\mu p_p^\nu q^\rho \frac{1}{q^2} \bar{u}(p_2) \gamma^\sigma v(p_1), \quad (2)$$

where $q = p_1 + p_2$, ϵ^μ is the polarization vector of the vector meson, and $f_{V \rightarrow P}(q^2)$ is the transition form factor, defined by

$$\langle P(p_p) | J_\sigma^{em} | V(p_v) \rangle = f_{V \rightarrow P}(q^2) \varepsilon_{\mu\nu\rho\sigma} \epsilon^\mu p_p^\nu q^\rho. \quad (3)$$

To calculate the form factor of $f_{V \rightarrow P}(q^2)$, we use the quark-flavor mixing scheme to express the vector mesons of ϕ and ω as two orthogonal flavor states of $|\psi_q\rangle$ and $|\psi_s\rangle$ with one mixing angle scenario, parameterized as [7, 8]

$$\begin{pmatrix} |\phi\rangle \\ |\omega\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta^V & -\sin \theta^V \\ \sin \theta^V & \cos \theta^V \end{pmatrix} \begin{pmatrix} |\psi_q\rangle \\ |\psi_s\rangle \end{pmatrix}, \quad (4)$$

where $|\psi_q\rangle = \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle$, $|\psi_s\rangle = |s\bar{s}\rangle$, and θ^V is the mixing angle, which has been extensively studied in the literature [7]. In this paper, we adopt its value to be $\theta^V \simeq -3.18^\circ$ in the quark-flavor basis. Under this scheme, the physical states of ω and ϕ can be written as combinations of $Q\bar{Q}$ states:

$$\begin{aligned} |\phi\rangle &= \frac{\cos \theta^V}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle - \sin \theta^V |s\bar{s}\rangle, \\ |\omega\rangle &= \frac{\sin \theta^V}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle + \cos \theta^V |s\bar{s}\rangle. \end{aligned} \quad (5)$$

For the outgoing pseudoscalar mesons, we use $|\pi^0\rangle = |u\bar{u} - d\bar{d}\rangle/\sqrt{2}$, while the states of η and η' can be written in terms of the two orthogonal states of $|\eta_q\rangle$ and $|\eta_s\rangle$ in the quark-flavor mixing scheme, given by [9, 10]

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{pmatrix}, \quad (6)$$

where $|\eta_q\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$, $|\eta_s\rangle = |s\bar{s}\rangle$, and β is the mixing angle constrained to be $\beta \simeq 37^\circ \sim 42^\circ$ [10]. Consequently, the valence states of $\eta^{(\prime)}$ can be presented as:

$$\begin{aligned} |\eta\rangle &= \cos \beta \frac{|u\bar{u} + d\bar{d}\rangle}{\sqrt{2}} - \sin \beta |s\bar{s}\rangle, \\ |\eta'\rangle &= \sin \beta \frac{|u\bar{u} + d\bar{d}\rangle}{\sqrt{2}} + \cos \beta |s\bar{s}\rangle. \end{aligned} \quad (7)$$

Combining with Eqs. (1), (3) and (5), the transition form factor of $V \rightarrow \pi^0$ can be found by summing up the relevant Fock states to be

$$f_{V \rightarrow \pi^0}(q^2) = \frac{\cos \theta^V}{\sqrt{2}} f_{|\psi_q\rangle \rightarrow |\pi^0\rangle}(q^2) = \frac{\cos \theta^V}{\sqrt{2}} f_{V_{q\bar{q}} \rightarrow \pi^0}(q^2). \quad (8)$$

Similarly, the transition from factors of $V \rightarrow \eta, \eta'$ have the forms

$$\begin{aligned} f_{\phi \rightarrow \eta} &= \cos \theta^V \cos \beta f_{|\psi_q\rangle \rightarrow |\eta_q\rangle} + \sin \theta^V \sin \beta f_{|\psi_s\rangle \rightarrow |\eta_s\rangle}, \\ f_{\phi \rightarrow \eta'} &= \cos \theta^V \sin \beta f_{|\psi_q\rangle \rightarrow |\eta_q\rangle} - \sin \theta^V \cos \beta f_{|\psi_s\rangle \rightarrow |\eta_s\rangle}, \\ f_{\omega \rightarrow \eta} &= \sin \theta^V \cos \beta f_{|\psi_q\rangle \rightarrow |\eta_q\rangle} - \cos \theta^V \sin \beta f_{|\psi_s\rangle \rightarrow |\eta_s\rangle}, \\ f_{\omega \rightarrow \eta'} &= \sin \theta^V \sin \beta f_{|\psi_q\rangle \rightarrow |\eta_q\rangle} + \cos \theta^V \cos \beta f_{|\psi_s\rangle \rightarrow |\eta_s\rangle}, \end{aligned} \quad (9)$$

where $f_{|\psi_q\rangle \rightarrow |\eta_q\rangle}$ and $f_{|\psi_s\rangle \rightarrow |\eta_s\rangle}$ can be replaced by $f_{V_{Q\bar{Q}} \rightarrow P_{Q\bar{Q}}}(Q = u, d, s)$.

In the LFQM, a neutral meson wave function is constructed by the simple structure of $Q\bar{Q}$ in terms of its constituent quark Q and anti-quark \bar{Q} with the total momentum p and spin S as [4],

$$\begin{aligned} |M(p, S, S_z)\rangle &= \int [dk_1][dk_2] 2(2\pi)^3 \delta^3(p - k_1 - k_2) \\ &\times \sum_{\lambda_1 \lambda_2} \Phi_M(k_1, k_2, \lambda_1, \lambda_2) b_Q^+(k_1, \lambda_1) d_{\bar{Q}}^+(k_2, \lambda_2) |0\rangle, \end{aligned} \quad (10)$$

where

$$[dk] = \frac{dk^+ d^2 k_\perp}{2(2\pi)^3}, \quad (11)$$

Φ_M is the amplitude of the corresponding $Q\bar{Q}$ and $k_{1(2)}$ ($\lambda_{1(2)}$) is the on-mass shell LF momentum (helicity) of the internal quark. In the momentum space, the wave function Φ_M can be expressed as a covariant form [2, 3]

$$\begin{aligned} \Phi_M(z, k_\perp) &= \left(\frac{k_1^+ k_2^+}{2[M_0^2 - (m_Q - m_{\bar{Q}})^2]} \right)^{\frac{1}{2}} \bar{u}(k_1, \lambda_1) \Gamma v(k_2, \lambda_2) \phi_M(z, k_\perp), \\ M_0^2 &= \frac{m_{\bar{Q}}^2 + k_\perp^2}{z} + \frac{m_Q^2 + k_\perp^2}{1-z}, \end{aligned} \quad (12)$$

where Γ stands for

$$\begin{aligned} \Gamma &= \gamma_5 \quad (\text{pseudoscalar}, S=0), \\ \Gamma &= -\not{\hat{\varepsilon}}(S_z) + \frac{\hat{\varepsilon} \cdot (k_1 - k_2)}{M_0 + m_Q + m_{\bar{Q}}} \quad (\text{vector}, S=1), \end{aligned} \quad (13)$$

and

$$\begin{aligned}\hat{\varepsilon}^\mu(\pm 1) &= \left[\frac{2}{p^+} \vec{\varepsilon}_\perp(\pm 1) \cdot \vec{p}_\perp, 0, \vec{\varepsilon}_\perp(\pm 1) \right], \quad \vec{\varepsilon}_\perp(\pm 1) = \mp(1, \pm i)/\sqrt{2}, \\ \hat{\varepsilon}^\mu(0) &= \frac{1}{M_0} \left(\frac{-M_0^2 + p_\perp^2}{p^+}, p^+, p_\perp \right).\end{aligned}\tag{14}$$

The LF relative momentum variables (z, k_\perp) are defined by

$$\begin{aligned}k_1^+ &= zp^+, \quad k_2^+ = (1-z)p^+, \\ k_{1\perp} &= zp_\perp - k_\perp, \quad k_{2\perp} = (1-z)p_\perp + k_\perp.\end{aligned}\tag{15}$$

In principle, the momentum distribution amplitude $\phi_M(z, k_\perp)$ can be obtained by solving the LF QCD bound state equation [2]. However, before such first-principle solutions are available, we would have to consider phenomenological ones. One example that has been widely used in the the Gaussian type wave function [5], given by

$$\phi_M(z, k_\perp) = N \sqrt{\frac{1}{N_c} \frac{dk_z}{dz}} \exp\left(-\frac{\vec{k}^2}{2\omega_M^2}\right),\tag{16}$$

where $N = 4(\pi/\omega_M^2)^{\frac{3}{4}}$, $\vec{k} = (k_\perp, k_z)$, and k_z is defined through

$$z = \frac{E_Q + k_z}{E_Q + E_{\bar{Q}}}, \quad 1-z = \frac{E_{\bar{Q}} - k_z}{E_Q + E_{\bar{Q}}}, \quad E_i = \sqrt{m_i^2 + \vec{k}^2}\tag{17}$$

by

$$k_z = \left(z - \frac{1}{2}\right) M_0 + \frac{m_{\bar{Q}}^2 - m_Q^2}{2M_0}, \quad M_0 = E_Q + E_{\bar{Q}}.\tag{18}$$

and $dk_z/dz = E_Q E_{\bar{Q}}/z(1-z)M_0$. From Eq. (3), one has

$$\begin{aligned}\langle P_{Q\bar{Q}}(p_p) | J_\sigma^{em} | V_{Q\bar{Q}}(p_v) \rangle &= N_c \int \frac{d^4 k_3}{(2\pi)^4} \Lambda_P \left\{ \text{Tr} \left[\left(-\not{\varepsilon} + \frac{\hat{\varepsilon} \cdot (k_1 - k_3)}{M_0 + m_Q + m_{\bar{Q}}} \right) \right. \right. \\ &\quad \left. \left. \frac{i(-\not{k}_3 + m_{\bar{Q}})}{k_3^2 - m_{\bar{Q}}^2 + i\epsilon} \not{\varepsilon}_\sigma \frac{i(\not{k}_2 + m_Q)}{k_2^2 - m_Q^2 + i\epsilon} \gamma^5 \frac{i(\not{k}_1 + m_Q)}{k_1^2 - m_Q^2 + i\epsilon} \right] \right\} \\ &\quad + (k_{1(3)} \leftrightarrow k_{3(1)}, m_Q \leftrightarrow m_{\bar{Q}}),\end{aligned}\tag{19}$$

The hadronic matrix element of the $V \rightarrow P$ transition can be newly parametrized in terms of the initial and final meson momenta, given by

$$\langle P_{Q\bar{Q}}(p_p) | J_\sigma^{em} | V_{Q\bar{Q}}(p_p + q) \rangle = f_{V_{Q\bar{Q}} \rightarrow P_{Q\bar{Q}}}(q^2) \varepsilon_{\mu\nu\rho\sigma} \epsilon^\mu p_p^\nu q^\rho,\tag{20}$$

where we have used the LF momentum variables (x, k_\perp) and worked in the frame that the transverse momentum is purely longitudinal, *i.e.*, $q_\perp = 0$. We note that $q^2 = q^+ q^- \geq 0$

covers the entire range of momentum transfers. Therefore, the trace in Eq. (19) can be easily carried out. The form factor $f_{V_{Q\bar{Q}} \rightarrow P_{Q\bar{Q}}}$ is then found to be

$$f_{V_{Q\bar{Q}} \rightarrow P_{Q\bar{Q}}}(q^2) = N_c \int_0^r dx \int \frac{d^2 k_\perp}{2(2\pi)^3} \frac{2x' \phi_V^*(x', k_\perp) \phi_P(x, k_\perp)}{m_Q^2 + k_\perp^2} \times \left\{ m_Q + \frac{1}{W_V} \left[r k_\perp^2 + (1-r) \left(2x M_{0P} k_z - \frac{x' k_\perp^2}{1-x'} \right) \right] \right\}, \quad (21)$$

where $N_c = 3$ is the number of colors and

$$x = r x', \quad r = \frac{m_V^2 + m_P^2 - q^2 + \sqrt{(m_V^2 + m_P^2 - q^2)^2 - 4m_V^2 m_P^2}}{2m_V^2}. \quad (22)$$

At $q^2 = 0$, the form factor in Eq. (21) is evaluated to be

$$f_{V_{Q\bar{Q}} \rightarrow P_{Q\bar{Q}}}(0) = N_c \int dx \int \frac{d^2 k_\perp}{2(2\pi)^3} \frac{2x \phi_V^*(x, k_\perp) \phi_P(x, k_\perp)}{m_Q^2 + k_\perp^2} \left(m_Q + \frac{k_\perp^2}{W_V} \right). \quad (23)$$

It has been noted [11] that in the purely longitudinal $q^+ > 0$ frame, the nonvalence contribution [12] to the form factor has to be included and the frame dependence should be checked [13]. To avoid the nonvalence contribution, one may use the Drell-Yan-West frame, *i.e.* $q^+ = 0$, in which it contains only the valence part [11]. In this frame, we obtain

$$f_{V_{Q\bar{Q}} \rightarrow P_{Q\bar{Q}}}(q^2) = N_c \int_0^1 dx \int \frac{d^2 k_\perp}{2(2\pi)^3} \frac{2x \phi_V^*(x, k_\perp) \phi_P(x, k'_\perp)}{m_Q^2 + k_\perp^2} \times \left\{ m_Q + \frac{2}{W_V} \left[k_\perp^2 + \frac{(k_\perp q_\perp)^2}{q^2} \right] \right\} \quad (24)$$

with $k_\perp = (x-1)q_\perp + k'_\perp$, which also agrees with Eq. (D2) in Ref. [11] and Eq. (4.13) in Ref. [14], respectively. At $q^2 = 0$, the formula for $f_{V_{Q\bar{Q}} \rightarrow P_{Q\bar{Q}}}(0)$ from Eq. (24) is the same as that in Eq. (23), which is a frame independent quantity as expected. For $q^2 \neq 0$, we remark that the results in the two frames should not be different too much in our cases of the light-to-light transitions [11].

The interaction between the photon and leptons is given by the conventional QED [6]. One easily obtains the differential decay rates normalized to the radiative decay widths of $V \rightarrow P\gamma$ as

$$\frac{d\Gamma(V \rightarrow P\ell^+\ell^-)}{\Gamma(V \rightarrow P\gamma)dq^2} = \frac{\alpha}{3\pi} \frac{1}{q^2} \left(1 - \frac{4m_\ell^2}{q^2} \right)^{1/2} \left(1 + \frac{2m_\ell^2}{q^2} \right) \quad (25)$$

$$\times \left[\left(1 + \frac{q^2}{m_V^2 - m_P^2} \right)^2 - \frac{4q^2 m_V^2}{(m_V^2 - m_P^2)^2} \right]^{3/2} \left| \frac{f_{V \rightarrow P}(q^2)}{f_{V \rightarrow P}(0)} \right|^2. \quad (26)$$

III. NUMERICAL RESULTS

To numerically calculate the transition form factor of $f_{V \rightarrow P}$, we need to specify the quark masses of $m_{u,d,s}$ and the parameters appearing in $\phi_M(x, k_\perp)$ ($M = V$ or P). Here, we have chosen the quark masses as $m_q = m_{u,d} = 0.25$ and $m_s = 0.42$ in GeV. To constrain the meson scale parameter of ω_M in Eq. (16), we use the branching ratio of $V \rightarrow P\gamma$, given by

$$\mathcal{B}(V \rightarrow P\gamma) = \frac{\alpha(M_V^2 - M_P^2)^3}{24M_V^3\Gamma_V} |f(0)_{V \rightarrow P}|^2. \quad (27)$$

Explicitly, we take [15]

$$\mathcal{B}^{exp}(\omega \rightarrow \pi^0\gamma) = (8.28 \pm 0.28) \times 10^{-2}, \quad (28)$$

$$\mathcal{B}^{exp}(\phi \rightarrow \pi^0\gamma) = (1.27 \pm 0.06) \times 10^{-3}. \quad (29)$$

which lead to $|f_{\omega \rightarrow \pi^0}(0)| = (2.297 \pm 0.05) \times 10^{-3}$ and $|f_{\phi \rightarrow \pi^0}(0)| = (1.33 \pm 0.038) \times 10^{-4}$ in MeV^{-1} , respectively. We can then fit the parameters of $w_{\omega,\phi}$ from Eq. (23) if we input the quark masses.

The numerical results for $F_\omega(q^2) \equiv f_{\omega \rightarrow \pi^0}(q^2)/f_{\omega \rightarrow \pi^0}(0)$ in the $q^+ > 0$ and $q^+ = 0$ frames of the LFQM are shown in Fig. 1. From the figure, we see that our results in the LFQM, particularly the one in $q^+ = 0$ fit well with the NA60 experimental data [1] even though they are slightly lower for the large q^2 region. Note that the numerical values in the $q^+ = 0$

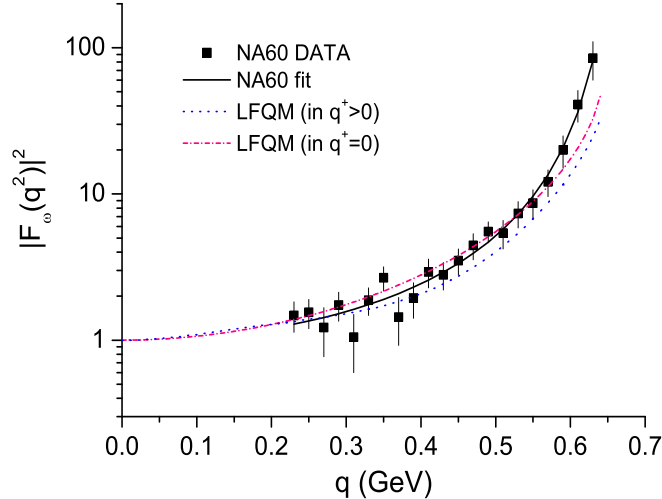


FIG. 1. (Color online) $F_\omega(q^2) \equiv f_{\omega \rightarrow \pi^0}(q^2)/f_{\omega \rightarrow \pi^0}(0)$ in the LFQM.

frame are slightly larger than those in $q^+ > 0$ in the large q^2 region. The difference may result from the nonvalence part in the $q^+ > 0$ frame [11]. In the following calculations, we will only use the form factors evaluated in the $q^+ = 0$ frame.

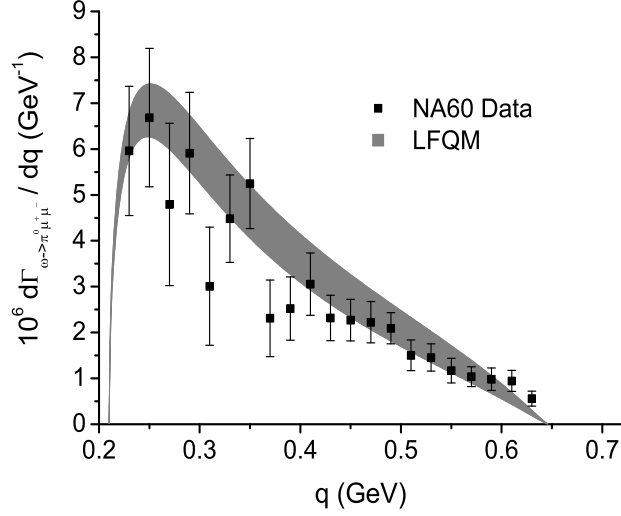


FIG. 2. Differential decay width of $\omega \rightarrow \pi^0 \mu^+ \mu^-$ as a function of q in the LFQM, where the small band is due to the uncertainty of the data in Eq. (27).

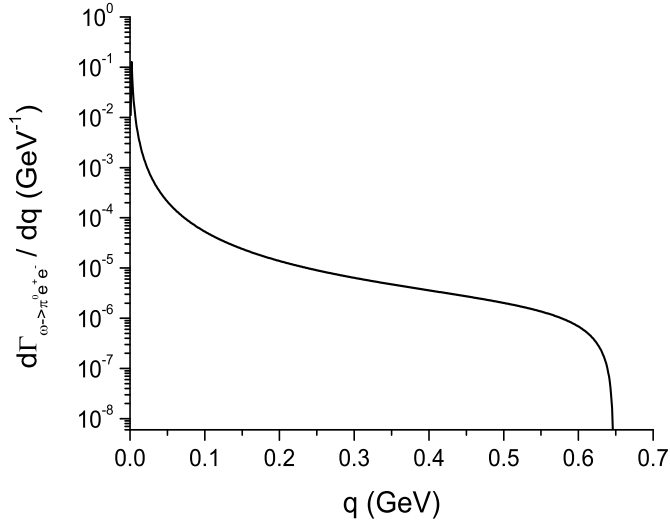


FIG. 3. Same as Fig. 2 but for the e mode.

As seen from Eq. (26), the differential decay widths of $V \rightarrow P \ell^+ \ell^-$ depend on the factor of $1/q^2$, highly suppressed by the phase space. In Figs. 2~5, we display the differential

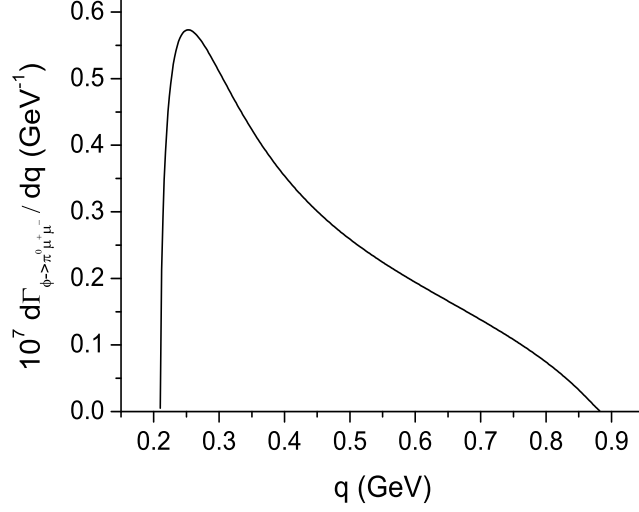


FIG. 4. Differential decay width of $\phi \rightarrow \pi^0 \mu^+ \mu^-$ as a function of q in the LFQM.

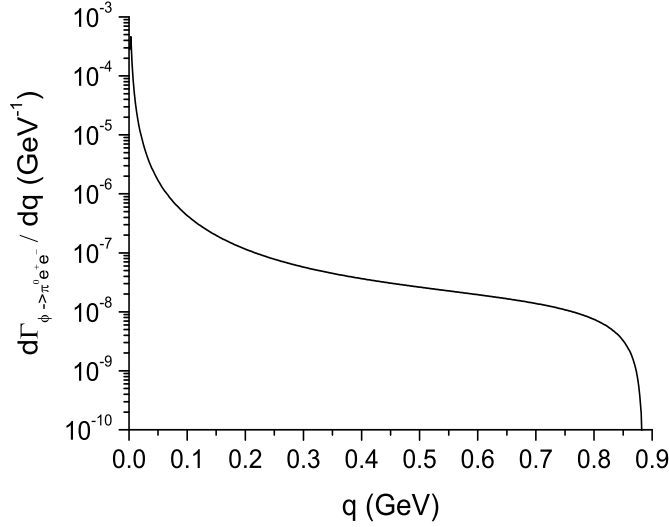


FIG. 5. Same as Fig. 4 but for the e mode.

decay widths of $V \rightarrow \pi^0 \ell^+ \ell^-$ ($V = \omega, \phi$ and $\ell = e, \mu$) as functions of q in the LFQM. From Fig. 2, we find that our result for $\omega \rightarrow \pi^0 \mu^+ \mu^-$ is consistent with the experimental data from NA60 [1]. Integrating over q^2 in Eq. (26), we can obtain the branching ratios of $V \rightarrow \pi^0 \ell^+ \ell^-$. Our results in the LFQM are shown in Table I. In the table, we also give the experimental data listed in PDG [15] as well as the theoretical predictions based on the vector meson method (VMD) [16] and dispersion relations (DR) [17]. From Table I, we

TABLE I. Decay branching ratios of $V \rightarrow \pi^0 \ell^+ \ell^-$ ($V = \omega, \phi$ and $\ell = e, \mu$) in the LFQM, where the experimental data listed in PDG [15] as well as the theoretical predictions based on the vector meson method (VMD) [16] and dispersion relations (DR) [17] are also given.

Model	LFQM	PDG [15]	VMD [16]	DR [17]
$10^4 \mathcal{B}_{\omega \rightarrow \pi^0 e^+ e^-}$	7.82 ± 0.39	7.7 ± 0.6	7.9	7.6...8.1
$10^4 \mathcal{B}_{\omega \rightarrow \pi^0 \mu^+ \mu^-}$	1.21 ± 0.15	1.3 ± 0.4	0.92	0.94...1.0
$10^5 \mathcal{B}_{\phi \rightarrow \pi^0 e^+ e^-}$	1.35 ± 0.12	1.12 ± 0.28	1.6	1.39...1.51
$10^6 \mathcal{B}_{\phi \rightarrow \pi^0 \mu^+ \mu^-}$	3.48 ± 0.57	—	4.8	3.7...4.0

observe that our predicted values for $\omega \rightarrow \pi^0 e^+ e^-$ and $\pi^0 \mu^+ \mu^-$ in the LFQM agree with those in PDG [15] in errors and also are close to the theoretical values in VMD [16] and DR [17], respectively. On the other hand, our results for the $\phi \rightarrow \pi^0 \ell^+ \ell^-$ decay modes are slightly smaller than those in Refs. [16, 17]. However, it is interesting to note that the decay branching ratio of $\phi \rightarrow \pi^0 e^+ e^-$ in the LFQM is consistent with the experimental data [15].

We now consider the processes of $V \rightarrow \eta^{(\prime)}$ ($V = \omega, \phi$). In Fig. 6, we show the transition form factor of $F_\phi \equiv f_{\phi \rightarrow \eta}(q^2)/f_{\phi \rightarrow \eta}(0)$ in the LFQM. In the figure, we also plot the experimental data from SND at the VEPP-2M collider [18]. Our result is consistent with the current data. Since the data contain large errors when $q > 0.2$ GeV, more precise experimental measurements are clearly needed to test our model. In Table II, we list all possible

TABLE II. Decay branching ratios of $V \rightarrow \eta^{(\prime)} \ell^+ \ell^-$ ($V = \omega, \phi$ and $l = e, \mu$), where we have also shown the experimental data in PDG [15] and the theoretical results based on the VMD [16] and large N_c [19] calculations.

Model	LFQM	PDG [15]	VMD [16]	Large N_c [19]
$10^6 \mathcal{B}_{\omega \rightarrow \eta e^+ e^-}$	3.22 ± 0.28	—	6.0	3.20 ± 0.10
$10^9 \mathcal{B}_{\omega \rightarrow \eta \mu^+ \mu^-}$	1.81 ± 0.23	—	1.8	1.00 ± 0.00
$10^4 \mathcal{B}_{\phi \rightarrow \eta e^+ e^-}$	1.07 ± 0.02	1.15 ± 0.10	1.1	1.09 ± 0.06
$10^6 \mathcal{B}_{\phi \rightarrow \eta \mu^+ \mu^-}$	6.86 ± 0.18	< 9.4	6.8	6.44 ± 0.69
$10^7 \mathcal{B}_{\phi \rightarrow \eta' e^+ e^-}$	2.97 ± 0.10	—	—	—

decay branching ratios of $V \rightarrow \eta^{(\prime)} \ell^+ \ell^-$ in the LFQM. In the table, we have also shown

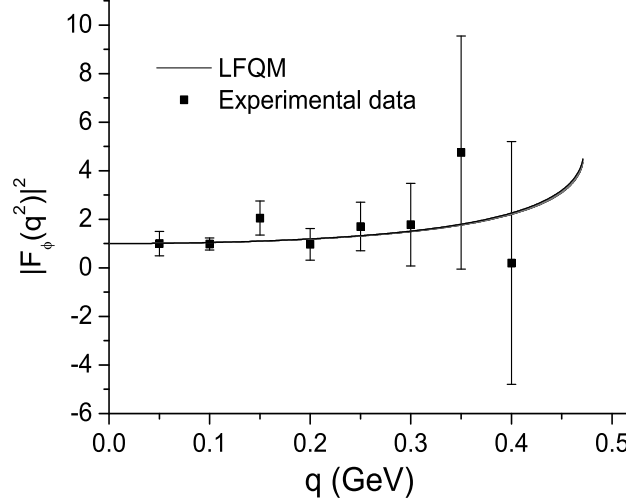


FIG. 6. $F_\phi(q^2) \equiv f_{\phi \rightarrow \eta}(q^2)/f_{\phi \rightarrow \eta}(0)$ in the LFQM.

the experimental data in PDG [15] and the theoretical results based on the VMD [16] and large N_c [19] calculations. Note that most of these decay modes have not been measured yet except $\phi \rightarrow \eta e^+ e^-$. From the table, we see that our results are close to those in the large N_c calculations [19] except the one for $\omega \rightarrow \eta \mu^+ \mu^-$. The prediction of $\omega \rightarrow \eta e^+ e^-$ in the VMD model is about two times larger than ours but those of $\omega \rightarrow \eta \mu^+ \mu^-$ and $\phi \rightarrow \eta \ell^+ \ell^-$ are consistent with the ones in the LFQM. Comparing with the only experimental value in $\phi \rightarrow \eta e^+ e^-$, our result fits well with the data in error.

IV. CONCLUSIONS

We have studied the form factors of the light vector mesons (ω and ϕ) into pseudoscalar mesons (π^0 , η and η') in the LFQM. In our calculations, we have adopted the Gaussian-type wave function and evaluated the form factors for the momentum dependences in all allowed q^2 regions. Our numerical results of $f_{\omega \rightarrow \pi^0}(q^2)/f_{\omega \rightarrow \pi^0}(0)$ are close to the experimental data by NA60. Similarly, our values for that of $\phi \rightarrow \eta e^+ e^-$ compare well with the data by SND at the VEPP-2M collider. With these form factors, we have calculated all possible decay branching ratios of $V \rightarrow P \ell^+ \ell^-$ with $V = \omega$ and ϕ , $P = \pi^0$, η and η' and $\ell = e$ and μ . Explicitly, we have found that our numerical results can fit with the data, such as those for $\omega \rightarrow \pi^0 \ell^+ \ell^-$ and $\phi \rightarrow \pi^0 e^+ e^-$ by NA60 and $\phi \rightarrow \eta e^+ e^-$ by SND. We also predict that the

branching ratios of $\phi \rightarrow \pi^0 \mu^+ \mu^-$, $\omega \rightarrow \eta e^+ e^-$, $\omega \rightarrow \eta \mu^+ \mu^-$, $\phi \rightarrow \eta \mu^+ \mu^-$, and $\phi \rightarrow \eta' e^+ e^-$ to be $(3.48 \pm 0.57) \times 10^{-6}$, $(3.22 \pm 0.28) \times 10^{-6}$, $(1.81 \pm 0.23) \times 10^{-9}$, $(6.86 \pm 0.18) \times 10^{-6}$ $(2.97 \pm 0.10) \times 10^{-7}$ respectively. Some of these modes could be measured in the future experiments.

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